

Supergravity before 1976¹

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Abstract

A story on how an attempt to realize W. Heizenberg idea that the neutrino might be a Goldstone particle had led in its development to the discovery of supergravity action.

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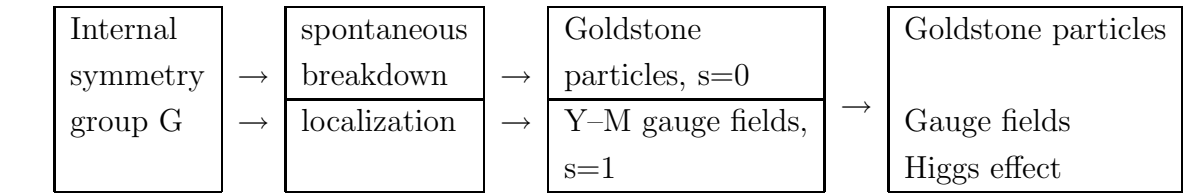
1 Introduction

There are two kinds of fields which are directly related to continuous symmetries. These are the Goldstone and Yang–Mills fields.

The Goldstone fields are a manifestation of spontaneously broken symmetries. The Yang – Mills fields appear when a symmetry is localized.

The both kinds of fields were predicted theoretically, and at the moment of their invention seemed to have no relation to reality.

However, revealed by Higgs and others, their mutual interplay exhibiting itself as the Higgs effect has thoroughly changed the situation. And nowadays the concept of the gauge fields and of the spontaneously broken symmetries is the backbone of theories unifying all known interactions.



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The modern idea of the unification involves not only internal symmetries but also a new type of symmetry discovered at the beginning of 70's, namely, supersymmetry, which is intrinsically intertwines with the former.

Main features of the supersymmetry have been discussed in the talks by P. Dimopolous and S. Fayet [1]. So, I shall remind only that supersymmetry was independently discovered by three groups of authors:

Yu. Gol'fand & E. Lichtman (1971)
D. Volkov & V. Akulov (1972)
J. Wess & B. Zumino (1974)

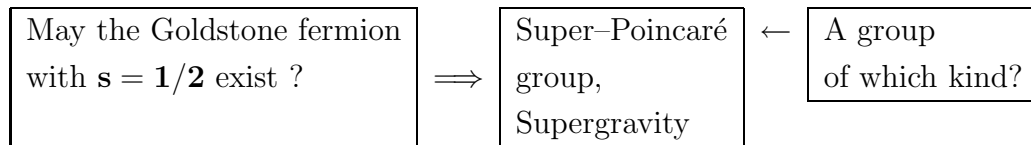
The motivation and starting points used by these groups were quite different.

The motivation of Gol'fand and Lichtman [2] was to introduce a parity violation in the quantum field theory. The starting point of the paper by Akulov and myself [3, 4] was the question whether Goldstone particles with spin one half might exist. Wess and Zumino [5] performed the generalization of the supergroup which first appeared in the Neveu–Schwarz–Ramond dual model [6, 7].

The approach of the Volkov–Akulov papers is the most appropriate for gauging the super–Poincaré group, which was done a little bit later in the papers by Soroka and myself [8, 9]. The last–mentioned papers are the natural continuation of the Volkov–Akulov papers since the transformation laws for gauge fields are determined by the same group structure as that used for the description of the Goldstone fields.

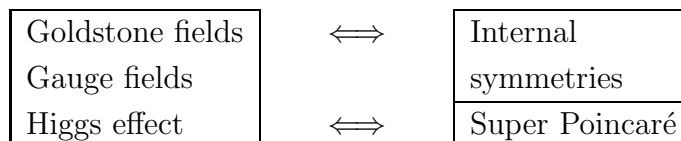
As it has already been mentioned, our approach was caused by the question about possible existence of the spin 1/2 Goldstone particles.

The second question closely related to the first one is: what is the group whose break-down yields Goldstone fields with the required properties? In some respect this is the reverse of the usual consideration of Goldstone and gauge fields when an internal symmetry group is given and the properties of the fields are derived from the known symmetry group properties.



Answering these questions led us to the discovery of the super–Poincaré group.

It is convenient to consider the internal symmetry groups and the super–Poincaré group in parallel since it gives the possibility of learning which features of the both cases are common and which are different.



As we shall see below the analogy between the two cases gives rather rich intuitive insight which helps one to trivialize the procedure of gauging the super-Poincaré group and getting the supergravity action.

Let us firstly recall that one of the main directions of the research in theoretical particle physics in the 60's was the Current Algebra initiated by M. Gell-Mann [10, 11, 12], and the PCAC hypothesis [13], which produced many interesting results in the field of weak and strong interactions, including a number of the so called soft pion, or low energy, theorems (Y. Nambu & D. Lurie [14], S. Adler [15], S. Weinberg [16],...). The essential progress was achieved when this direction was goldstonized (Goldstone [17], Nambu, Jona-Lozinio [18]), and especially when S. Weinberg [19] and J. Schwinger [20] proposed the method of Phenomenological Lagrangians which easily reproduced all soft pion theorems with any number of pions.

At the time of the XIVth Conference on High Energy Physics (Vienna, 1968) the problem of the Current Algebra and of the Phenomenological Lagrangians was intensively discussed (see Weinberg's rapporteur talk [21]). There were two papers presented in the current algebra section of the conference in which the generalization of the method of Phenomenological Lagrangians to an arbitrary internal symmetry group had been elaborated. One paper was presented by B. Zumino [22] (co-authors C. Callan, S. Coleman and J. Wess), and another one by myself [23]. The main results of the papers were practically identical. The difference was that in Ref. [23]² the works of E. Cartan on symmetric spaces and his method of the exterior differential forms were intensively used.

The general procedure for constructing Phenomenological Lagrangians for Goldstone particles of internal symmetry groups [22, 23] is the following.

A group G is factorized

$$G = KH, \tag{1}$$

where H is a stationary subgroup of G , and K is the coset space $K=G/H$.

The g -algebra valued differential 1-forms

$$G^{-1}dG = H^{-1}(K^{-1}dK)H + H^{-1}dH$$

represent the vielbeins $(G^{-1}dG)_k$ and the connection 1-forms $(G^{-1}dG)_h$, where k and h are subspaces of the g -algebra corresponding, respectively, to K and H .

The Phenomenological Lagrangian is constructed out of the vielbeins:

$$L = \frac{1}{2}Sp(G^{-1}dG)_k(G^{-1}dG)_k, \tag{2}$$

while the connection forms are used to include interaction with other particles which belong to representations of the group G . G -multiplets are reduced to H -multiplets with respect to the stationary subgroup H and considered as independent in the approach

²The papers [23, 25] are contained in [26].

in question. The splitting of the G-multiplets into the H-multiplets together with the appearance of Goldstone fields is the main feature of the Phenomenological Lagrangian method.

A little bit later this procedure was generalized by J. Wess and B. Zumino [24] by introducing the so called Wess–Zumino term, and by me for the case of spontaneously broken symmetry groups containing the Poincaré group as a subgroup [25, 26]³. The Phenomenological Lagrangian for Goldstone fermions, which we now turn to consider, is an example of the latter generalization.

2 Goldstone Fermions, super–Poincaré group and its spontaneous breaking

Now we can return to the question how one can generalize the Poincaré group to the super–Poincaré group by requiring Goldstone particles to have spin one half. As it has been explained in the previous section, the quantum numbers of the Goldstone particles coincide with the quantum numbers of the K–generators. Therefore, to ensure the appearance of the Goldstone fermions, the Poincaré group should be generalized in such a way that the generalized version contains generators with spin one–half obeying commutation relations corresponding to the Fermi statistics. From a technical point of view the problem was what representation of the Poincaré group is the most appropriate for such generalization.

The solution to this technical problem was that the following representation of the Poincaré group had all required properties:

$$\mathbf{G}_{\text{Poincare}} = \begin{pmatrix} 1 & iX \\ 0 & 1 \end{pmatrix} \begin{pmatrix} L & 0 \\ 0 & (L^+)^{-1} \end{pmatrix}, \quad (3)$$

where $X_{\alpha\dot{\beta}}$, L_{α}^{β} and $(\mathbf{L}^+)^{-1}_{\dot{\beta}}^{\dot{\alpha}}$ are $\mathbf{2} \times \mathbf{2}$ matrices.

In the generalization to the super–Poincaré group $K_{\text{transl.}}$ plays the main role.

Let us write it down in a form consisting of four blocks:

$$K = \begin{pmatrix} 1 & iX \\ 0 & 1 \end{pmatrix} \quad (4)$$

Separating the blocks as follows

$$K' = \begin{pmatrix} 1 & - & iX \\ 0 & 1 & - \\ 0 & 0 & 1 \end{pmatrix} \quad (5)$$

³In [25], which was prepared in autumn, 1971 there is the first mentioning of the generalization of the Poincaré group to the super–Poincaré group.

one can insert into the newly formed hatched blocks Grassmann spinors θ_α and $\bar{\theta}_{\dot{\alpha}}$ so that K' becomes

$$K' = \begin{pmatrix} 1 & \theta & iX' \\ 0 & 1 & \bar{\theta} \\ 0 & 0 & 1 \end{pmatrix}. \quad (6)$$

The matrices K' form a group, but only under the condition that X' is complex. To match the reality condition for X' with that of X in eq.(4) and to conserve the group properties of (6) one should represent X' in the form

$$iX' = iX + \frac{1}{2}\theta\bar{\theta}.$$

The resulting expression for the super-Poincaré group is as follows

$$G_{SP} = \begin{pmatrix} 1 & \theta & iX + \frac{1}{2}\theta\bar{\theta} \\ 0 & 1 & \bar{\theta} \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} L & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & (L^+)^{-1} \end{pmatrix}. \quad (7)$$

In [3, 4] more general case of N-extended super-Poincaré group was considered. In this case θ -spinors acquire additional internal symmetry index and, in the second factor in (7) the unit matrix is replaced by an internal symmetry group matrix.

From (7) one gets the transformation law for the superspace coordinates:

$$X' = X + i\bar{\theta}\gamma\epsilon, \quad \theta' = \theta + \epsilon, \quad \bar{\theta}' = \bar{\theta} + \bar{\epsilon} \quad (8)$$

as well as the following expressions for the left-invariant superspace vielbein one-forms

$$e^a = dX^a - i\bar{\theta}\gamma^a d\theta \quad (9)$$

$$\mathbf{e}^\alpha = \mathbf{d}\theta^\alpha. \quad (10)$$

The latter are obtained as components of $K^{-1}dK$ corresponding to the generators of K now being the supertranslation subgroup of (7).

The action for the Goldstone fermions is the pullback of a target superspace differential four-form onto the four-dimensional Minkovski subspace (the world space)

$$S = \frac{1}{24}\varepsilon_{abcd}e^ae^be^ce^d. \quad (11)$$

In eq. (11) as well as in further formulas exterior product is implied.

The action (11) gives the first example of a method widely applied nowadays for constructing superparticle, superstring and supermembrane actions out of the target superspace vielbein forms with their pulling back onto the worldsheet.

Now we can see what are the differences between (11) and (2). Firstly, the Lagrangian (11) is constructed not out of the 1-forms (10), as it may be expected, but out of the

1-forms (9). Secondly, the Lagrangian (11) is not the metric of a Riemann space as in (2), but the 4-volume form.

Interactions of the Goldstone fermions with particles of spin 0, $\frac{1}{2}$ and 1 were considered in [4, 27] with the use of the method developed in [25, 26]. Notwithstanding the differences from the internal symmetry case mentioned above, all soft Goldstone particle theorems were reproduced, and we became sure that our generalization of the Poincaré group may have something to do with reality. Our first paper was entitled “Is the neutrino a Goldstone particle?”, but since gauging the Poincaré group is directly connected with the Einstein–Cartan theory of gravity, the same should take place for any generalization of the Poincaré group. Therefore, our paper was concluded by the following sentence

‘...the gravitational interaction may be included by means of introducing the gauge fields for the Poincaré group. Note that if the gauge field for the fermionic transformation is also introduced, then as a result of the Higgs effect the massive gauge field with spin three-halves appears and the considered Goldstone particle with spin one-half disappears...’

which was the first may be somewhat implicit mentioning of the theory of supergravity as a theory containing the Einstein–Cartan action and the action for the Rarita–Schwinger field, the latter being massless in the absence of the spontaneous supersymmetry breaking. We also considered the possibility that the super–Poincaré group might be an approximate symmetry so that (in analogy with π , ρ and A' mesons in hadron physics [16]) Goldstone particles with nonzero mass and the Higgs effect coexisted.

3 Gauged super–Poincaré group

Now let us go over to the procedure of gauging super–Poincaré group.

Supersymmetry is spontaneously broken symmetry. Therefore, it is convenient to consider its gauged version from the very beginning in the form containing Goldstone fermion fields.

Note that the 1-forms (9) as well as (10) are not covariant with respect to the local transformations of the group under consideration. But if local transformation laws of Goldstone and gauge fields *are correlated*

$$K' = G_L K H,$$

$$A'(d) = G_L A(d) G_L^{-1} + G_L d G_L^{-1},$$

the sum

$$\tilde{A}(d) = K^{-1} dK + K^{-1} A(d) K$$

transforms as

$$\tilde{A}'(d) = H \tilde{A}(d) H^{-1} + H d H^{-1}.$$

Only $\langle \tilde{\mathbf{A}}(\mathbf{d}) \rangle_{\mathbf{h}}$ are the gauge fields, and $\langle \tilde{\mathbf{A}}(\mathbf{d}) \rangle_{\mathbf{k}}$ are non-gauge fields which have absorbed the Goldstone degrees of freedom and transform covariantly under subgroup H.

The differential forms presented above can be used as blocks for constructing invariants of the local group, and then summing up all independent invariants one gets the general form of the gauge field action. It is self-evident that the action constructed this way contains not only the terms with spontaneously broken symmetry but a gauge invariant action as well. The latter can be easily extracted by counting the physical degrees of freedom. In other words, one should recall that the basic essence of the Higgs effect is that the Goldstone degrees of freedom though absorbed by the gauge fields are physical degrees of freedom. If some combination of the local invariants considered as an action results in two physical gauge field degrees of freedom, it means that corresponding Goldstone fields do not contribute and they either drop out of the action or become auxiliary fields. The latter, as shown in the next section, is the case for N=1 supergravity.

Here we remind what methods for considering the Higgs effect in the case of internal symmetry groups were used at the end of 60's and at the beginning of 70's. Such reminding is useful for the reader to better understand the state of the development of theoretical physics at that time, and, as it has been already said, the consideration of internal symmetries and supersymmetry in parallel helps one to formulate and to solve the problems in the latter case.

In the case of internal symmetry groups the approach for considering the Higgs effect may be roughly divided on to the following stages

- the method described above for constructing the action out of the differential forms with the correlated transformation law of the gauge and Goldstone fields. The advantage of this method is its generality, the drawback is its phenomenological nature which exhibits itself in the presence of a number of arbitrary constants;
- studying simplified models constructed for understanding possible mechanisms of spontaneous symmetry breakdown;
- studying realistic gauge field theories coupled to matter fields which can ensure spontaneous symmetry breaking.

All these stages were applied and are being applied now to supersymmetric field theories.

The gauge fields for the N=1 super-Poincaré group are described by a g_{susy} -valued matrix

$$A_{SUSY}(d) = \begin{pmatrix} \omega(d) & \psi(d) & e(d) \\ 0 & 0 & \bar{\psi}(d) \\ 0 & 0 & \bar{\omega}(d) \end{pmatrix}$$

where $\omega(d)$ is the Lorentz connection, $\psi(d)$ is the Rarita-Schwinger gauge field and $e(d)$ is the vierbein 1-form.

Corresponding $\tilde{A}_{SUSY}(d)$ forms are

$$\begin{aligned}\tilde{e}(d) &= e(d) + DX + i \left[(2\psi(d) + D\theta)\bar{\theta} - \theta(2\bar{\psi}(d) + D\bar{\theta}) \right], \\ \tilde{\psi}(d) &= \psi(d) + D\theta, \quad \tilde{\omega}(d) = \omega(d), \\ \tilde{R}(d, d') &= R(d, d') = d\omega(d') - d'\omega(d) + [\omega(d), \omega(d')].\end{aligned}\tag{12}$$

where D is the covariant differential with the connection form $\omega(d)$

4 Supergravity action and the super–Higgs effect

Contracting the indices of the differential forms (12) one can construct the following locally invariant differential four–forms [8, 9]:

$$W_1 = \tilde{R}(d_1, d_2)\tilde{e}(d_3)\tilde{e}(d_4) - \text{Einstein–Cartan action}$$

$$W_2 = D\tilde{\psi}(d_1, d_2)\tilde{e}(d_3)\tilde{\psi}(d_4) - \text{Rarita–Schwinger kinetic term}$$

$$W_3 = \tilde{e}(d_1)\tilde{e}(d_2)\tilde{e}(d_3)\tilde{e}(d_4) - \text{cosmological term}$$

$$W_4 = \tilde{\psi}(d_1)\tilde{e}(d_2)\tilde{e}(d_3)\tilde{\psi}(d_4) - \text{Rarita–Schwinger mass term}$$

The resulting action for N=1 supergravity is the sum

$$S = S_1 + S_2\tag{13}$$

with

$$S_1 = a_1 W_1 + a_2 W_2\tag{14}$$

being the pure SUGRA action and

$$S_2 = a_3 W_3 + a_4 W_4\tag{15}$$

being the terms which arise due to the spontaneous breakdown of the super–Poincaré group. Let us stress once again that each of the terms W_i and, hence, action S_1 and S_2 obey local supersymmetry.

The action S_1 (14) is the same as one which is now accepted as the action of $N = 1$ supergravity, and since the action for any newly proposed dynamical system completely determines all its physical properties, the time, when the action was firstly written down is the date of the discovery of the theory. So N=1 supergravity as a physical theory was discovered in 1973. At the XVII International Conference on High Energy Physics (London, 1974) B. Zumino told [28]:

‘Volkov and Soroka [3] have developed a description of curved superspace which combined gravitational theory with interaction of particles of spin 3/2, 1 and 1/2. Can a theory of this kind, because of compensation of divergences due to supersymmetry, provide a renormalizable description of gravitational interaction?’

Now we proceed to explain (13-15) by counting the physical degrees of freedom of the gauge fields. It is self-evident that due to the fact that all terms are differential 4-forms the gravitational field has two physical components and corresponds to the unbroken symmetry. The Rarita-Schwinger field corresponds to the broken or unbroken local supersymmetry if it has four or two physical degrees of freedom, respectively. Thus, using this simple argument of counting the degrees of freedom one can find out whether supersymmetry is broken or not. This depends on the values of the coefficients a_i in (13). To determine the coefficients it is sufficient to consider particular cases, the simplest examples being the Minkowski and the anti-de-Sitter space (if the mass term with a definite ratio of the mass to the curvature of the anti-de-Sitter space is added) as backgrounds for the Rarita-Schwinger field, and then go over to more general cases. It can be shown quite easily that if a background Einstein field satisfies the equation of motion for the term W_1 the Rarita-Schwinger field, in this background, has only two physical components, the Goldstone fermions do not contribute to the first two terms in the sum (13) and all effects of the spontaneous breaking of the super-Poincaré group are contained only in the third and the fourth term. If in (13) a_i are arbitrary the Rarita-Schwinger field has four degrees of freedom, so in the general case the Goldstone fermions are physical degrees of freedom, which is the realization of the super-Higgs effect.

The auxiliary Goldstone fermion fields (as well as X^a) can be excluded from the action S_1 without changing its physical contents. But, firstly, this procedure is not unique, and, secondly, which is more important, the off-mass-shell local invariance of S_1 is lost and reduced to the local invariance on the mass shell with all its unpleasant consequences that the algebra is not closed, structure “constants” depend on fields etc. And since on the mass shell the Lorentz connection $\omega(d)$ becomes a function of $e(d)$ and $\psi(d)$ its transformation law changes and becomes rather complicated. The action S_1 , with the local supersymmetry held on the mass shell, was regained in 1976 [29, 30]. In the review [31] (p.319) the main result of [30] was formulated as follows:

‘Deser and Zumino [30] have shown that supergravity is nothing but the spin 3/2 Rarita – Schwinger Lagrangian minimally coupled to Cartan first order formalism of general relativity.’

As to the terms S_2 (15) in (13), they are reproduced in all N=1 supergravity theories with matter fields specifically added to make supersymmetry spontaneously broken. Of

course, in such theories the constants a_3 and a_4 are determined by the choice of the matter fields, their masses and interaction constants.

There is a number of papers where the equivalence of our approach to the conventional one is established. See, for example, [38] and references therein.

Our approach can be applied to any $N \leq 8$ case as well. In papers [8, 9] we wrote that we considered “the simplest possible local invariants”. In $N = 1$ case the local invariants W_i constitute the complete set. When N increases the number of local invariants also increases. If $N > 2$ not only gauge and Goldstone fields, but also all non-gauge fields which are the superpartners of the gauge fields in the supergravity multiplet, should be used for the construction of the complete set of invariants. The situation is the same as in the case of spontaneously broken internal symmetry with the only difference that in the case of supersymmetry gauge and non-gauge fields may belong to the same supermultiplet.

5 Superspace formulation of Supergravity (First steps into Superspace)

The first superspace formulation of supergravity was attempted by R. Arnowitt, P. Nath and B. Zumino [32]. They proposed the generalization of the Einstein 4-dimensional action to superspace with coordinates (x, θ) in the following form

$$S = \int R \sqrt{Ber g} d^4 \theta d^4 x.$$

The analysis of this action revealed that it contains a number of drawbacks such as

- a) the presence of fermionic ghosts,
- b) the action does not admit “flat” superspace as a solution;
- c) for any solution torsion is zero, which is a consequence of Riemann geometry chosen;
- d) the holonomy group of the tangent space was $OSp(3,1/4)$ which is too large.

Upon realizing these drawbacks two groups of authors [33, 34] generalized the E. Cartan’s method of differential geometry to superspace and argued that

- the holonomy group for superspace curvature is the Lorentz group;
- flat superspace has torsion and, hence, torsion has to be included into the theory of supergravity.

These points are now part of all existing versions of supergravity. The intensive development of the superspace approach to supergravity has been carried out starting from 1976 and up to now. At the first stages Wess and Zumino, Ogievetski and Sokatchev, Siegel, Ferrara and Ro  ek, Scherk and many others have made significant contribution to this development.

6 Résumé

Before 1976 the following general ideas had been proposed and the following results obtained

- Super-Poincaré group ($N=1$, and $N > 1$)
- Flat superspace, supervielbeins.
- Notion of Goldstone fermions
- Phenomenological Lagrangian for Goldstone fermions interacting with particles of $s=0, \frac{1}{2}$ and 1
- Gauging the Super-Poincaré group
- Supergravity action, $N=1$ (the off-mass-shell formulation).
- Super-Higgs effect, $N \geq 1$
- First attempts to the superspace formulation of supergravity.

As concerns the techniques developed, it is mostly connected with the application of E. Cartan's methods of the exterior differential forms and differential geometry. E. Cartan's methods were applied to the component formalism for constructing the Goldstone fermion and supergravity action, and to trace a way to the superspace formulation of supergravity.

Nowadays E. Cartan's methods, being the fundamentals of the fiber bundle theory, are among the main tools used in theoretical physics.

As the speakers at the Conference were asked to present not only their ideas and results but also to describe the path along which they went to reach them, I shall describe some of the main impulses and motivation which helped my coauthors and me to pass the way to supersymmetry and supergravity.

Of the greatest importance was, as I call it, my personal inclination to the problem of connection between the spin and statistics, and to related topics. My first papers had been done in that direction of research.

My PhD thesis was devoted to the calculation of radiative corrections to some effects in the quantum electrodynamics of scalar particles. I had performed it following J. Schwinger papers, and from that time I started to consider him as my teacher by correspondence and, as his student by correspondence, I carefully studied all his papers at stock.

On my only personal meeting with J. Schwinger in Kiev in 1959 we had a fruitful discussion during which I proposed a possible generalization of an admissible class of variations in Schwinger's quantum dynamical principle by connecting the variations with the symmetry properties of the action. A little bit later, J. Schwinger published a paper [35] in which the generalization proposed was elaborated. For me this episode became as an exam which I successfully passed to my teacher by correspondence. Listening the talk by S. Glashow at the Erice Conference I envied his lucky fortune to be in everyday contact with J. Schwinger.

Of course, the use of Grassmann variables as variations of fermion fields in the Schwinger quantum variation principle was an essential step to supersymmetry, while the Rarita-Schwinger field was a step to supergravity. I am sure that J. Schwinger had been quite prepared to write down the supergravity action in the form of (13) in the middle of 60's or even earlier, and only his engaging in studying other problems can explain why he had not done it.

At the end of 50's W. Heisenberg and W. Pauli proposed a non-linear theory which claimed to become the base for understanding the spectra and interactions of elementary particles. A little bit later, W. Pauli changed his role as the coauthor for the role of the most severe Heisenberg's opponents⁴. But W. Heisenberg continued his work, and, trying to overcome the mathematical difficulties of the theory, used his great intuition to find the place for each of the existing particles in the scheme. He succeeded in this attempt for a number of particles. Considering the neutrino, W. Heisenberg proposed that it might be a Goldstone particle connected with the spontaneous breakdown of parity [37]. The assumption impressed me immensely, but at that time I had not been prepared to implement this idea.

As I mentioned in section 2 my approach to the Phenomenological Lagrangians was different in some points from other approaches. The difference was that the starting point of [23] was the action (2) with an arbitrary metric tensor on which no symmetry requirements were imposed. Calculating the on-mass-shell scattering amplitudes I got that they depend only on manifestly covariant quantities such as the curvature tensor and its covariant derivatives. Upon imposing the simplest of possible restrictions, namely, that the covariant derivative of the curvature tensor vanishes

$$R_{absd,f} = 0$$

I became aware that this condition is the definition of the symmetric spaces proposed by E. Cartan. I began to study his works and they opened for me the mathematical beauty of differential geometry and taught many lessons on how one might apply it to physical theories.

⁴W. Heisenberg describes this period in a monograph [36]

In previous sections I have stressed the role of Goldstone, Nambu, Schwinger, Weinberg, Higgs and others whose papers introduced me into the world of nonlinear realized (secret) symmetries.

So the way to the discovery of supersymmetry and supergravity was rather long and not a straight line. And now I see that the seeds of supersymmetry had been thrown onto the fertile soil and beautiful mathematical structures had grown out of them, and all of us, theorists and experimentalists, are waiting for the time when we can touch and taste the fruit.

I am very grateful to the Organizing Committee of the Conference for giving me this unique opportunity to illuminate the period in the development of supergravity which is usually omitted exposing the history of its discovery.

References

- [1] See talks in these Proceedings.
- [2] Yu. Gol'fand and E. Lichtman, *JETP Letters* **13** (1971) 323.
- [3] D. Volkov and V. Akulov, *JETP Letters* **16** (1972) 438. *Phys. Lett.* **46B** (1973) 109.
- [4] V. Akulov and D. Volkov, *Theor. Math. Phys.* **18** (1974) 28.
- [5] J. Wess and B. Zumino, *Nucl. Phys.* **B70** (1974) 39.
- [6] J.-L. Gervais and B. Sakita, *Nucl. Phys.* **B34** (1971) 633.
- [7] A. Neveu and J. Schwarz, *Nucl. Phys.* **B31** (1971) 86. P. Ramond, *Phys. Rev. D* **3** (1971) 2415.
- [8] D. Volkov and V. Soroka, *JETP Letters* **18** (1973) 312.
- [9] D. Volkov and V. Soroka, *Theor. Math. Phys.* **20** (1974) 829.
- [10] M. Gell-Mann, *Physics* **1** (1964) 63.
- [11] M. Gell-Mann, see a talk in these Proceedings.
- [12] D. Gross, see a talk in these Proceedings.
- [13] Y. Nambu, *Phys. Rev. Lett.* **4** (1960) 380. I. Bernstein et al., *Nuov. Cim.* **17** (1960) 757.

- [14] Y. Nambu and D. Lurie, *Phys. Rev.* **125** (1962) 1429.
- [15] S. Adler, *Phys. Rev.* **137B** (1965) 1022; *ibid.* **139B** (1965) 1638.
- [16] S. Weinberg, *Phys. Rev. Lett.* **18** (1967) 507.
- [17] J. Goldstone, *Nuov. Cim.* **19** (1961) 155.
- [18] Y. Nambu and G. Yona-Losinio, *Phys. Rev.* **122** (1961) 345.
- [19] S. Weinberg, *Phys. Rev.* **166** (1968) 1568.
- [20] J. Schwinger *Phys. Lett* **24B** (1967) 173.
- [21] S. Weinberg, Proceedings of the XIV Conference on High Energy Physics, Vienna, 1968.
- [22] S. Coleman, J. Wess and B. Zumino, *Phys. Rev.* **177** (1969) 2239;
C. Callan et. al. *Phys. Rev.* **177** (1969) 2247.
- [23] D. Volkov, Kiev preprint ITF-69-75, 1969.
- [24] J. Wess and B. Zumino, *Phys. Lett.* **37B** (1971) 95.
- [25] D. Volkov, International Seminar on Functional Methods in QFT and Statistics.
Preprint of Lebedev Institute (PhIAN) n 141, Moscow, 1971, 4.
- [26] D. Volkov, *Soviet J. Part. Nucl.* **4** (1973) 1.
- [27] V. Akulov and D. Volkov, *JETP Lett.* **17** (1973) 261.
- [28] B. Zumino in Proceedings of the XVII Conference on High Energy Physics, London, 1974.
- [29] S. Ferrara, D. Freedman and P. Van Nieuwenhuizen, *Phys. Rev.* **D13** (1976) 3214.
- [30] S. Deser and B. Zumino, *Phys. Lett.* **62B** (1976) 335.
- [31] P. Fayet and S. Ferrara, *Phys. Rep.* **32** (1977) 249.
- [32] R. Arnowitt, P. Nath and B. Zumino, *Phys. Lett.* **56B** (1975) 81.
- [33] B. Zumino, Proc. Conf. on Gauge Theories and Modern Field Theory, Northeastern Univ., 1975, p.255.
- [34] V. Akulov, D. Volkov and V. Soroka, *JETP Lett.* **22** (1975) 187.
- [35] J. Schwinger, *Proc. Nat. Acad. Sci.* **47** (1961) 1075.

- [36] W. Heizenberg, *Der Teil und Das Ganze*, Munchen, 1969.
- [37] W. Heizenberg, *Introduction to the unified theory of elementary particles*. Interscience Publishers, London, New-York, Sidney, 1966.
- [38] E. Ivanov and A. Kapustnikov, *Nucl. Phys.* **333** (1990) 439.
- [39] E. Cartan, *Bull. Soc. Math. France*, **54** (1926) 214; **55** (1927) 114.